

What you will learn about:
Adding and Subtracting Polynomials

Types of Polynomials

POLYNOMIALS

polynomial—A monomial, or two or more monomials combined by addition or subtraction, is a polynomial.

- **monomial**—A polynomial with exactly one term is called a monomial.
- **binomial**—A polynomial with exactly two terms is called a binomial.
- **trinomial**—A polynomial with exactly three terms is called a trinomial.

Here are some examples of polynomials:

Polynomial	$b + 1$	$4y^2 - 7y + 2$	$4x^6 + x^5 + 8x^2 - 9x + 1$
Monomial	14	$8y^2$	$-9x^3y^3$
Binomial	$a + 7$	$4b - 5$	$y^2 - 16$
Trinomial	$x^2 - 7x + 12$	$9y^2 + 2y - 8$	$6m^4 - m^3 + 8m$

Notice that every monomial, binomial, and trinomial is also a polynomial. They are just special members of the "family" of polynomials and so they have special names. We use the words **monomial**, **binomial**, and **trinomial** when referring to these special polynomials and just call all the rest **polynomials**.

No $+$ / $-$ →

EXAMPLE 6.1

Determine whether each polynomial is a monomial, binomial, trinomial, or other polynomial.

- ⓐ $4y^2 - 8y - 6$ **trinomial**
- ⓑ $-5a^4b^2$ **monomial**
- ⓒ $2x^5 - 5x^3 - 9x^2 + 3x + 4$ **Polynomial**
- ⓓ $13 - 5m^3$ **binomial**
- ⓔ q **monomial**

Degree of a polynomial

Degree of a term

Sum of exponents

Degree of a constant

zero

Degree of a polynomial

Highest power of all terms

Find the degree of the following polynomials.

- ⓐ $10y$ **D = 1**
- ⓑ $4x^3 - 7x + 5$ **D = 3**
- ⓒ -15 **D = 0**
- ⓓ $-8b^2 + 9b - 2$ **D = 2**
- ⓔ $8xy^2 + 2y$ **D = 3**

Adding and Subtracting
Polynomials

Preform the indicated operation:

$$12q^2 + 9q^2$$

$$21q^2$$

$$-15c^4 + 8c^4$$

$$-7c^4$$

$$8m - (-5m)$$

$$13m$$

$$(5y^2 - 3y + 15) + (3y^2 - 4y - 11)$$

$$8y^2 - 7y + 4$$

$$(9w^2 - 7w + 5) - (2w^2 - 4)$$

$$7w^2 - 7w + 9$$

$$(2x^2 - 3xy - 2y^2) + (5x^2 - 3xy)$$

$$7x^2 - 2y^2$$

$$(a^2 + b^2) - (a^2 + 5ab - 6b^2)$$

$$-5ab + 7b^2$$

$$(a^3 - a^2b) - (ab^2 + b^3) + (a^2b + ab^2)$$

$$a^3 - b^3$$

$$(x^3 - x^2y) + (xy^2 + y^3) - (x^2y + xy^2)$$

$$x^3 - 2x^2y + y^3$$

Evaluate $5x^2 - 8x + 4$ when

x=4

$$\begin{aligned}5(4)^2 - 8(4) + 4 \\5(16) - 32 + 4 \\80 - 32 + 4 = 52\end{aligned}$$

x=-2

$$\begin{aligned}5(-2)^2 - 8(-2) + 4 \\5(4) + 16 + 4 \\20 + 16 + 4 = 40\end{aligned}$$

x=0

$$5(0)^2 - 8(0) + 4 = 4$$

The polynomial $h = -16t^2 + 250$ gives the height of a ball t seconds after it is dropped from a 250 foot tall building. Find the height after 2 seconds. After 4 seconds.

$$\begin{aligned}h &= -16(2)^2 + 250 \\&= -16(4) + 250 \\&= -64 + 250 \\&= 186\end{aligned}$$

$$\begin{aligned}h &= -16(4)^2 + 250 \\&= -16(16) + 250 \\&= -256 + 250 \\&= -6\end{aligned}$$

The polynomial $6x^2 + 15xy$ gives the cost, in dollars, of producing a rectangular container whose top and bottom are squares with sides x feet and sides of height y feet. Find the cost of producing a box that is 4 feet wide and six feet tall.

$$x=4 \quad y=6$$

$$\begin{aligned}6(4)^2 + 15(4)(6) \\6(16) + 15(4)(6) \\96 + (60)(6) \\96 + 360 \\456\end{aligned}$$

What you will learn about:
Multiplication Properties of Exponents

Simplify Expressions with Exponents

Simplify:

$$\begin{array}{rcl} 2^5 & & 21^1 \\ 1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 & & 21 \\ & & 32 \\ -5^2 = -25 & & (-5)^2 = 25 \end{array}$$

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$$

$$(0.218)^2$$

$$\begin{array}{r} 1 \\ 16 \\ .218 \\ \hline 1744 \\ 2180 \\ \hline 43600 \\ .047524 \end{array}$$

Simplify Expressions Using the Product Property of Multiplying Exponents

$$a^n \cdot a^m = a^{n+m}$$

Simplify:

$$y^5 \cdot y^6 = y^{11}$$

$$b^9 \cdot b^8 = b^{17}$$

$$2^3 \cdot 2^8 = 2^{11}$$

$$d^4 \cdot d^5 \cdot d^2$$

$$d^{11}$$

$$t^6 \cdot t^2 \cdot t$$

$$t^9$$

Simplify Expressions Using the Power Property of Exponents

$$(a^m)^n = a^{m \cdot n}$$

$$\begin{array}{l} (a^2)^3 \\ a^2 \cdot a^2 \cdot a^2 \end{array}$$

$$\begin{array}{l} \text{Simplify: } (y^5)^9 \\ y^{45} \end{array}$$

$$\begin{array}{l} (5^4)^3 \\ 5^{12} \end{array}$$

Simplifying Expressions Using
the Product to a Power
Property

$$(ab)^m = a^m \cdot b^m$$

Simplify:

$$\frac{(-9d)^2}{81d^2}$$

$$\frac{(2wx)^5}{32w^5x^5}$$

$$\frac{(-3y)^3}{-27y^3}$$

$$(y^3)^6(y^5)^4$$

$$y^{18}y^{20}$$

$$(-6x^4y^5)^2$$

$$36x^8y^{10}$$

$$(-2c^4d^2)^3$$

$$-8c^{12}d^6$$

$$\frac{(5m)^2(3m^3)}{(25m^2)(3m^3)}$$

$$75m^5$$

$$\frac{(2n)^3(3n^{10})}{(8n^3)(3n^{10})}$$

$$24n^{13}$$

$$(3x^2)(-4x^3)$$

$$-12x^5$$

$$(5y^7)(-7y^4)$$

$$-35y^{11}$$

$$(-6b^4)(-9b^5)$$

$$54b^9$$

$$\left(\frac{5}{6}x^3y\right)(12xy^2)$$

$$10x^4y^3$$

$$\left(\frac{2}{5}a^4b^3\right)(15ab^3)$$

$$6a^5b^6$$

$$\left(\frac{2}{5}r^5s\right)(12r^6s^7)$$

$$\frac{24}{5}r^{11}s^8$$

What you will learn about:
Multiplication of Polynomials

Multiply:

$$4(x + 3) \quad -2x(x - 4)$$

$$\underline{4x+12}$$

$$\underline{-2x^2+8}$$

$$(3t^2 - 9)t$$

$$\underline{3t^3-9t}$$

$$5x(x + 4y) \quad 2p(6p + r)$$

$$\underline{5x^2+20xy}$$

$$\underline{12p^2+2pr}$$

$$-2y(y^2 + 3y - 5)$$

$$\underline{-2y^3-6y^2+10y}$$

$$2x^3(x^2 - 8x + 1)$$

$$\underline{2x^5-16x^4+2x^3}$$

$$-6a^3(3a^2 - 2a + 6)$$

$$\underline{-18a^5+12a^4-36a^3}$$

FOIL

F - First

O - Outer

I - Inner

L - Last

$$\begin{array}{c} F \quad L \\ (y + 5)(y + 8) \\ y^2 + \cancel{8y} + 5y + 40 \end{array}$$

$$y^2 + 13y + 40$$

$$(5x - 9)(4x - 3)$$

$$20x^2 - 15x - 36x + 27$$

$$(2y - 5)(3y + 4)$$

$$6y^2 + 8y - 15y - 20$$

$$(3x - y)(2x - 5)$$

$$6x^2 - 15x - 2xy - 5y$$

$$(n^2 + 4)(n - 1)$$

$$n^3 - n^2 + 4n - 4$$

$$(2ab + 5)(4ab - 4)$$

$$8a^2b^2 - 8ab + 20ab - 20$$

$$8a^2b^2 + 12ab - 20$$

$$(b+3)(2b^2 - 5b + 8)$$

$$\begin{array}{r} 2b^3 - 5b^2 + 8b \\ 6b^2 - 15b + 24 \\ \hline \end{array}$$

$$2b^3 + b^2 - 7b + 24$$

$$(2x-3)(4x^2 - 6x + 7)$$

$$\begin{array}{r} 8x^3 - 12x^2 + 14x \\ - 12x^2 + 18x - 21 \\ \hline \end{array}$$

$$8x^3 - 24x^2 + 32x - 21$$

$$(3t+1)(2t^2 - 4t - 5)$$

$$\begin{array}{r} 6t^3 - 12t^2 - 15t \\ 2t^2 - 4t - 5 \\ \hline \end{array}$$

$$6t^3 - 10t^2 - 19t - 5$$

What you will learn about:
 Multiplication of Polynomials
 Special Products

Binomial Squared

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Multiply:

$$(x+5)^2$$

$$(x+5)(x+5)$$

$$\begin{array}{r} x^2 \\ + 5x + 5x + 25 \\ \hline x^2 + 10x + 25 \end{array}$$

$$(y - 3)^2$$

$$y^2 - 6y + 9$$

$$(4x + 6)^2$$

$$16x^2 + 48x + 36$$

$$(2x - 3y)^2$$

$$4x^2 - 12xy + 9y^2$$

$$(4u^3 + 1)^2$$

$$16u^6 + 8u^3 + 1$$

$$(3y^2 - 2)^2$$

$$9y^4 - 12y^2 + 4$$

Conjugate Pair

$$(a + b), (a - b)$$

Multiply:

$$(x - 8)(x + 8)$$

$$x^2 + 6x - 8x - 64$$

$$x^2 - 64$$

$$(y + 3)(y - 3)$$

$$y^2 - 3y + 3y - 9$$

$$y^2 - 9$$

$$(3 - 5x)(3 + 5x)$$

$$9 + 15x - 15x - 25x^2$$

$$9 - 25x^2$$

$$(4 - 7y)(4 + 7y)$$

$$16 + 28y - 28y - 49y^2$$

$$16 - 49y^2$$